OCR
Oxford Cambridge and RSA

Friday 20 May 2016 – Morning

AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4725/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

1 Find $\sum_{r=1}^{n}(3r+1)(r-1)$, giving your answer in a fully factorised form. **[5]**

2 The complex number z has modulus $2\sqrt{3}$ and argument $-\frac{1}{3}\pi$. Giving your answers in the form $x+\mathrm{i}y$, where x and y are exact real numbers, and showing clearly how you obtain them, find

(i) z, **[2]**

(ii) $\dfrac{1}{(z^*-5\mathrm{i})^2}$. **[5]**

3 The quadratic equation $kx^2+x+k=0$ has roots α and β.

(i) Write down the values of $\alpha+\beta$ and $\alpha\beta$. **[1]**

(ii) Find the value of $\left(\alpha+\dfrac{1}{\alpha}\right)\left(\beta+\dfrac{1}{\beta}\right)$ in terms of k. **[5]**

4 The matrices **A**, **B** and **C** are given by **A** = $(a \;\; 2 \;\; 3)$, **B** = $(b \;\; 0 \;\; 5)$ and **C** = $\begin{pmatrix} 6 \\ 4 \\ -1 \end{pmatrix}$. Find

(i) $5\mathbf{A}-3\mathbf{B}$, **[2]**

(ii) **BC**, **[2]**

(iii) **CA**. **[2]**

5 The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 5 \text{ and } u_{n+1} = 3u_n + 2 \text{ for } n \geqslant 1.$$

Prove by induction that $u_n = 2\times 3^n - 1$. **[4]**

6 In an Argand diagram the points A and B represent the complex numbers $5+4\mathrm{i}$ and $1+2\mathrm{i}$ respectively.

(i) Given that A and B are the ends of a diameter of a circle C, find the equation of C in complex number form. **[4]**

The perpendicular bisector of AB is denoted by l.

(ii) Sketch C and l on a single Argand diagram. **[2]**

(iii) Find the complex numbers represented by the points of intersection of C and l. **[3]**

7 The matrix $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$ represents a transformation P.

(i) Describe fully the transformation P. **[2]**

The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} -3 & -1 \\ -1 & 0 \end{pmatrix}$.

(ii) Given that **M** represents transformation Q followed by transformation P, find the matrix that represents the transformation Q and describe fully the transformation Q. **[6]**

8 **(i)** Show that $\frac{1}{2r+1}-\frac{1}{2r+3}\equiv\frac{2}{(2r+1)(2r+3)}$. **[1]**

(ii) Hence find $\sum_{r=1}^{n}\frac{1}{(2r+1)(2r+3)}$, giving your answer as a single fraction. **[6]**

(iii) Find $\sum_{r=n}^{\infty}\frac{1}{(2r+1)(2r+3)}$, giving your answer as a single fraction. **[3]**

9 **(i)** The matrix $\mathbf{X}$ is given by $\mathbf{X}=\begin{pmatrix}a & 3 & -2\\ 0 & a & 5\\ 1 & 2 & 1\end{pmatrix}$. Show that the determinant of $\mathbf{X}$ is $a^2-8a+15$. **[3]**

(ii) Explain briefly why the equations

$$\begin{aligned}3x+3y-2z&=1\\ 3y+5z&=5\\ x+2y+z&=2\end{aligned}$$

do not have a unique solution and determine whether these equations are consistent or inconsistent. **[3]**

10 **(i)** Use an algebraic method to find the square roots of the complex number $9+40\mathrm{i}$. **[6]**

(ii) Show that $9+40\mathrm{i}$ is a root of the quadratic equation $z^2-18z+1681=0$. **[1]**

(iii) By using the substitution $z=\frac{1}{u^2}$, find the roots of the equation $1681u^4-18u^2+1=0$. Give your answers in the form $x+\mathrm{i}y$, where x and y are real. **[4]**

END OF QUESTION PAPER

Question		Answer	Marks	Guidance
1.		$\frac{1}{2}n(n+1)(2n+1)-n(n+1)-n$ $\frac{1}{2}n(2n+3)(n-1)$ or $n(n+\frac{3}{2})(n-1)$	M1* A1 A1 DM1 A1 **[5]**	Expand and attempt to use standard series, at least one used correctly Any two terms correct, may be unsimplified All terms correct Attempt to find 3 factors Obtain correct answer
2	(i)	$x=2\sqrt{3}\cos(-\frac{\pi}{3}), y=2\sqrt{3}\sin(-\frac{\pi}{3}), x^2+y^2=12, y=-x\sqrt{3}$ $\sqrt{3}-3\,\mathrm{i}$	M1 A1 **[2]**	Correct trig expression for x or y allow a positive angle or 2 equations for x and y, not involving trig Obtain correct answer as a complex number, extra answers not rejected gets A0
	(ii)	$\sqrt{3}+3\mathrm{i}$ $-1-(4\sqrt{3})\mathrm{i}$ $-\frac{1}{49}+\frac{4\sqrt{3}}{49}\mathrm{i}$	B1ft M1 A1 M1 A1 **[5]**	Correct conjugate seen, ft from their z in (i) Expand denominator Correct value seen Attempt to rationalise Obtain correct answer a.e.e.f. **N.B.** if 2 answers given in (i) award marks for better solution in (ii)

Question		Answer	Marks	Guidance
3.	(i)	$\alpha+\beta=-\frac{1}{k}, \alpha\beta=1$	B1 [1]	State correct values
	(ii)	*Either* $\alpha\beta+\frac{1}{\alpha\beta}+\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$ *Or* $(\alpha+\beta)(\beta+\alpha)$ $\frac{1}{k^2}$ or k^{-2}	M1 M1 A1 M2 A1 M1 A1 [5]	Expand expression Use correct process for $\alpha^2+\beta^2$ Obtain correct expression State $\alpha=\frac{1}{\beta}$ and $\beta=\frac{1}{\alpha}$ and substitute into given expression Obtain correct expression Use their value(s) in their expression Obtain correct single term answer
4.	(i)	$(5a-3b \quad 10 \quad 0)$	B1 B1 [2]	2 elements correct, must be a 1 × 3 matrix (not coordinates) 3rd element correct
	(ii)	$(6b-5)$	M1 A1 [2]	Single value Correct answer, must be a matrix
	(iii)	$\begin{pmatrix} 6a & 12 & 18 \\ 4a & 8 & 12 \\ -a & -2 & -3 \end{pmatrix}$	M1 A1 [2]	Obtain a 3 x 3 matrix All elements correct
5.		$3(2\times3^n-1)+2$	B1 M1 A1 B1 [4]	Show clearly result true for $n = 1$, accept $(u_1) = 2 \times 3 - 1 = 5$ Substitute for u_n in recurrence relation Establish correct result for u_{n+1} convincingly Clear statement of induction conclusion, provided 1st 3 marks earned

Question		Answer	Marks	Guidance
6.	(i)	3 + 3i $\sqrt{5}$ $\|z-3-3\mathrm{i}\| = \sqrt{5}$	B1 B1 M1 A1ft **[4]**	Obtain centre as a complex number, allow (3, 3) but not (3, 3i), give if correct in locus equation Obtain correct radius a.e.f. condone decimals Use correct form for locus Obtain correct answer from their centre and radius, $z = x + \mathrm{i}y$ is acceptable
	(ii)	Circle, centre in 1st quadrant, not touching or intersecting axes Straight line with –ve slope through (3, 3)	B1 B1 **[2]**	Centre need not be (3, 3) (3, 3) may be implied by working from (i) must be longer than the diameter. N.B. No circle drawn, B1 for l may be earned
	(iii)	2 + 5i 4 + i	M1 A1 A1 **[3]**	Gradient of l is – 2 used or attempt to solve Cartesian equations for C and l M0 if equations for C and/or l contain i. Obtain correct answers, must be complex numbers

Question		Answer	Marks	Guidance
7.	(i)	Shear, x-axis invariant and e.g. (0, 1) → (3, 1)	B1 B1 **[2]**	Must be a shear, otherwise 0/2. For invariant only allow parallel to or along x-axis, in x direction, for image allow 0.322^c, 18.4^o, $\tan^{-1}(1/3)$ or the complement, ignore scale factor if all OK otherwise. Column vectors for coordinates OK
	(ii)	*Either* $\begin{pmatrix}1 & 3\\0 & 1\end{pmatrix}\begin{pmatrix}a & b\\c & d\end{pmatrix}=\begin{pmatrix}-3 & -1\\-1 & 0\end{pmatrix}$ or **PQ** = **M** $\begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix}$ reflection in $y=-x$ *Or* $\begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix}$	B1 M1 A2 B2 **[6]** B2 B2 B2	Obtain correct matrix equation Sensible attempt at multiplication of a pair of 2× 2 matrices Obtain correct answer, A1 for 3 elements correct Must be describing Q If they say M = QP → $P^{-1}M = Q$, they can only get (possibly) M1 B2 if they obtain "correct" matrix for Q Diagram showing at least unit square and image under **M** coordinates shown State reflection in $y = -x$ Correct matrix

Question		Answer	Marks	Guidance
8.	(i)	$\frac{2r+3-(2r+1)}{(2r+1)(2r+3)}$ *or* $\frac{2r+3-2r-1}{(2r+1)(2r+3)} = \frac{2}{(2r+1)(2r+3)}$	B1 **[1]**	Establish **given** result correctly, might be a "0 = 0" type verification, or partial fractions
	(ii)	1/3 – 1/5 and 1/5 – 1/7 1/(2*n* + 1) – 1/(2*n* +3) $\frac{n}{3(2n+3)}$	M1 M1 A1 A1 M1 A1 **[6]**	Express terms as differences using (i) Attempt this for at least 1st two and last term First two terms correct, do not penalise missing factor of 2 Last term correct, do not penalise missing factor of 2, allow in terms of e.g. r or k Show correct cancelling Obtain a correct single fraction (denominator could be expanded) must be in terms of n. N.B. Be on look out for f(r) – f(r + 1) approach
	(iii)	*Either* $\frac{1}{6} - \frac{n-1}{3(2n+1)}$ $\frac{1}{2(2n+1)}$ *Or* $\frac{1}{(2n+1)}$ $\frac{1}{2(2n+1)}$	M1 A1 A1 **[3]** M1 A1 A1	Use sum to ∞ – sum to *n* -1 or *n* Obtain correct unsimplified expression, do not penalise missing factor of 2 Obtain a correct single fraction (denominator could be expanded) Start differences at *r* = *n* Obtain correct remaining term Obtain a correct single fraction (denominator could be expanded)

Question		Answer	Marks	Guidance
9.	(i)	$a^2 - 8a + 15$	M1 M1 A1 **[3]**	Show correct expansion process for 3 × 3 condone sign errors Correct expansion for any 2 × 2, may use Cramer's rule which is M2 Obtain **given** answer correctly
	(ii)	det **X** = 3^2 – 3 x 8 +15 =0 e.g. $3y+5z=5, or 3x-7z=-4, or 5x+7y=5$	B1 M1 A1 **[3]**	And must state not a unique solution or equivalent Attempt to solve equations Find a repeated equation and state consistency N.B. They may solve the equations first then deduce consistent and non-unique and gets 3/3 (possibly)
10.	(i)	$x^2 - y^2 = 9, \quad xy = 20$ ±(5 + 4i)	M1 A1 M1 M1 A1 A1 **[6]**	Equate real and imaginary parts of 9 + 40i and $(x + iy)^2$ Obtain correct equations a.e.f., e.g. allow 2ixy = 40i Rearrange to obtain quadratic in x^2 or y^2 Solve quadratic in x^2 or y^2 and square root Obtain one correct answer as a complex number Obtain other answer as complex number a.e.f. ±5 ± 4i is OK for A2, ±(5 ± 4i) scores A1 only
	(ii)		B1 **[1]**	Use sum and product of roots or solve equation or substitute into equation
	(iii)	$u = \frac{1}{5+4i}$ $\pm(\frac{5}{41} + \frac{4}{41}i), \ \pm(\frac{5}{41} - \frac{4}{41}i)$	B1 M1 A1 A1 **[4]**	Use substitution to obtain one of the **equations** from other **equation**, (must have = 0) Use result from (i) (either root) Obtain any 2 correct answers Obtain other 2 correct answers **S.C.**: If they solve quadratic for u^2 directly, use scheme above, M1 when they have used (i) to get a value for u.